Eavesdropping/Jamming of Communication Networks

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Gainesville, FL

Project #: FA9550-05-1-0137

Outline

1. Thanks
   - Organizations Involved
   - Collaborators
2. Wireless Network Jamming Problem
   - Motivation & Assumptions
   - Jamming Under Uncertainty
   - Other Formulations
3. Current Developments
   - Upper and Lower Bounds
   - Heuristic for Uncertain Case

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Organizations

- Air Force Office of Scientific Research
- Air Force Research Laboratory, Munitions Directorate, Eglin AFB
- European Office of Aerospace Research and Development
- University of Florida Research and Engineering Education Facility (REEF)

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Researchers Involved

Collaborators

- Clayton W. Commander, AFRL/MNGN and UF ISE
- Valeriy Ryabchenko, UF ISE
- Oleg Shylo, UF ISE
- Grigory Zrazhevsky, Kiev University

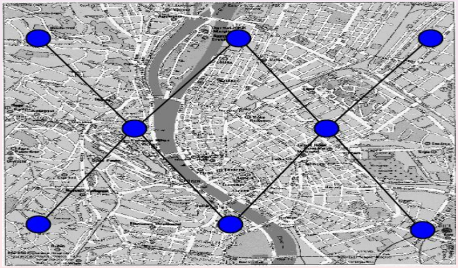
Outline

Problem Background

- The problem was motivated by AFRL/MNGN
- Military operations rely heavily on communication via wired & wireless telecom networks
- The ability to intercept/supress information flow in the network will provide a competitive edge over the adversary

Intuition

Find locations for minimum number of jamming devices to supress information flow on the network



Assumptions About Nodes and Jamming Devices

Equipped with omni-directional antennas

Jamming effectiveness $e(i, j)$ is decreasing function of distance from jammer j to node i

$$e(i,j) = \frac{\lambda}{R^2(i,j)}, \quad R(i,j) = \text{distance between node } i \text{ and device } j$$

$$\lambda \in \mathbb{R}. \text{ WLOG, let } \lambda = 1$$

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Definition

A node N is jammed if the cumulative energy received from all jammers exceeds some threshold E:

$$\sum_j \frac{1}{R^2(N,j)} \geq E. \tag{1}$$

This condition can be rewritten:

$$\sum_j \frac{1}{R^2(N,j)} \geq \frac{1}{L^2}, \quad \text{where } L = \sqrt{1/E} \tag{2}$$

Interpretation

Any jammer covers all points in a circle of radius L.

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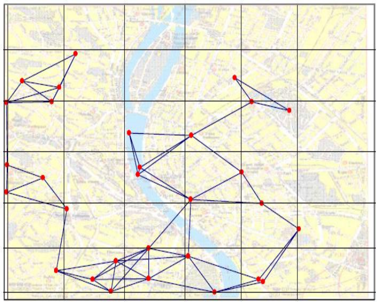
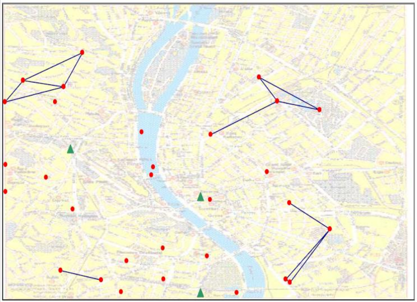
Definitions

Definition

A connection (arc) between two communication nodes is considered jammed if any of the two nodes is covered

Example





Outline

Continuous Formulation

No Information About Network

Let n be the number of jammers used. Given a region containing the network, say a square region that is $a \times a$, the problem is

$$\begin{aligned} & \text{Minimize } n \\ \text{s.t.} \quad & \sum_{i=1}^{n} \frac{1}{(u_i - x)^2 + (v_i - y)^2} \geq \frac{1}{L^2} \\ & \forall\, (x, y) : 0 \leq x \leq a, 0 \leq y \leq a, \end{aligned}$$

where (u_i, v_i) are the coordintates of jammer i.

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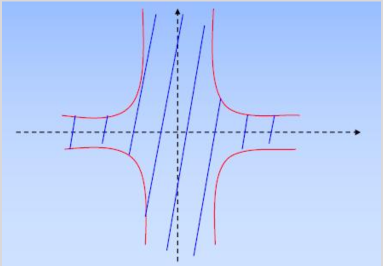
This problem is highly non-convex

Example

It is easy to see that the solution of the inequality:

$$\frac{1}{x^2} + \frac{1}{y^2} \geq C$$

represents an unbounded cross-shaped region in the (x, y) plane.



Integer Programming Approximation

No Information About Network

Let $X = \{X_1(u_1, v_1), \ldots, X_n(u_n, v_n)\}$ be a set of possible jammer locations. The optimization problem is:

$$\text{Minimize} \sum_{i}^{n} x_i$$

$$\text{s.t.} \quad \sum_{i=1}^{n} \frac{x_i}{(u_i - x)^2 + (v_i - y)^2} \geq \frac{1}{L^2}$$

$$\forall\, (x, y) : 0 \leq x \leq a, 0 \leq y \leq a$$

$$x_i \in \{0, 1\}$$

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Information Known About Network

Optimal Network Covering

- Given node locations
- Given potential jammer locations
- OBJECTIVE: Cover all nodes using minimal number of jammers

Connectivity Index Formulation

- Given network topology
- Given potential jammer locations
- OBJECTIVE: Place jammers such that *connectivity index is* $\leq C$

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Extensions and Complexity

Incorporation of Percentile Constraints

- Value at Risk (VaR)
- Conditional Value at Risk (CVaR)

Computational Complexity

All formulations are $\mathcal{NP}$-hard

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Jamming Under Complete Uncertainty

Our setup

- Usually interdiction efficiency determined by fraction of covered nodes/arcs
- We use no specific criterium because we consider the case of complete uncertainty
- We have NO information about node coordinates or the network topology
- The only reasonable approach is to jam all points in the area containing the network

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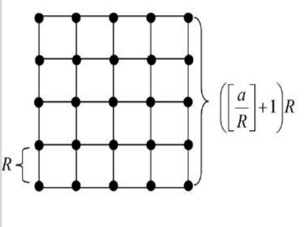
Problem Setup Cont.

Considered Formulation

Since finding the global optimal solution is hard, we consider covering a square of side a with jammers located at nodes of a uniform grid. The optimal solution for this class is a grid with largest step R covering the square. Problem is still non-trivial!

Example (jamming devices located at nodes of grid)

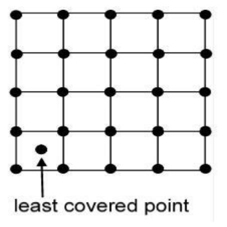


The Goal

We are seeking upper $\overline{R}$ and lower $\underline{R}$ bounds for the optimal grid step size R^* : $\underline{R} < R^* < \overline{R}$.

Lemma

For any covering of a square with a uniform grid, a point which receives the least amount of jamming energy lies inside a corner grid cell.



Lower Bound

Theorem

The unique solution of the equation

$$\frac{1}{2R^2}\left(\pi \ln\left(\frac{a}{R}+1\right)+\pi-3\right)=\frac{1}{L^2} \tag{3}$$

is a lower bound $\underline{R}$ *for the optimal grid step size* R^*.

Can be solved easily using numerical procedure, i.e. binary search, because (3) is monotonic.

Quality of Bound

Compare to Optimal Covering of Square with Circles

- Our LB $\Rightarrow$ number of jammers does not exceed $N_1 = (\frac{a}{R} + 2)^2$
- Kershner (1939) proved that in the limit, the minimum number of circles to cover area a^2 is $N_2 = \frac{2a^2}{3\sqrt{3L^2}}$
- To compare, consider $\frac{N_2}{N_1} = \frac{2x^2}{3\sqrt{3}} \frac{1}{(1+\frac{2x}{k^2})^2}$, where $x = \frac{R}{L}$ and $k = \frac{a}{L}$.

Rewrite (3) in terms of x and k

$$\frac{1}{x^2}(\pi \ln(\frac{k}{x}+1)+\pi-3)=2 \tag{4}$$

Example (solve for various value of k)

k	x	$\frac{N_2}{N_1}$
10^2	2.44	2.3
10^4	3.54	4.8
10^6	4.40	7.5
10^8	5.14	10.2

To see advantage of uniform grid over naive approach...

We prove that

$$\lim_{a\to\infty}\frac{N_2}{N_1}=\infty$$

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Upper Bound

Theorem

The solution of the equation

$$\frac{1}{R^2}\left(\frac{\pi}{2}\ln\left(\frac{2a}{R}+1\right)-\frac{1}{6(\frac{a}{R}+1)}+\frac{\pi}{2}+\frac{19}{3}\right)=\frac{1}{L^2} \qquad (5)$$

is an upper bound $\overline{R}$ *of the optimal grid step size* R^*.

- Function is monotone $\Rightarrow$ has unique solution
- $\overline{R}$ does not cover least jammed point (in corner grid)

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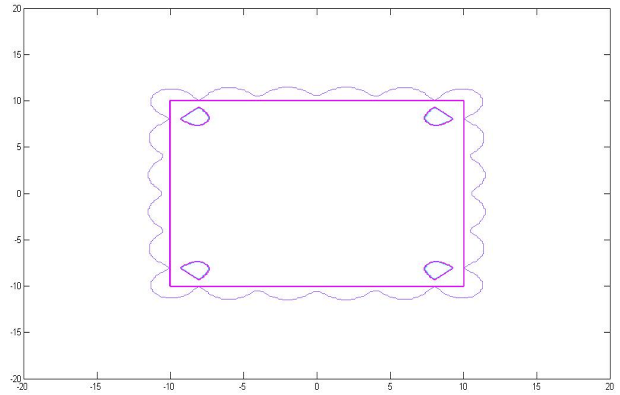
Example (Holes represent uncovered points)



Theorem

Convergence Result

$$\lim_{a \to \infty} \frac{\overline{R}}{\underline{R}} = 1,$$

where $\overline{R}$ *and* $\underline{R}$ *are bounds obtained from (5) and (3), correspondingly. Moreover, the following inequality holds:*

$$1 \leq \frac{\overline{R}}{\underline{R}} \leq \sqrt{1 + \frac{c}{\ln(a)}},$$

for $M, c \in \mathbb{R}$, *where* $\overline{R} > M$.

Outline

Heuristic for General Problem

Randomized Local Search

- Begin with random distribution of jamming devices
- Let S be a set of local minimums (i.e. the set of the least covered points)
- The quality of the solution is defined as a sum of jamming levels at the points from S
- (Repeat until solution is locally optimal)
  - Determine the least covered point from S
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Advantages:

- Can be used for the region of any shape
- Can be used to determine the best possible jamming of the given area by a certain number of jamming devices
- The jamming devices can have different properties
- Can be used for the non-uniform jamming (i.e. when some areas should be jammed more then the others)

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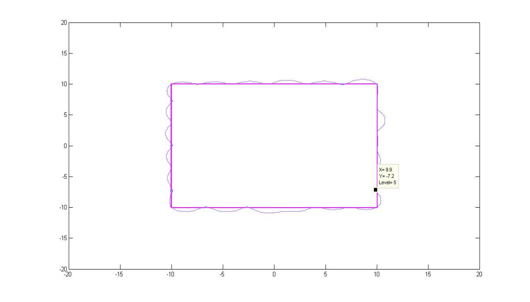
Computational experiments

Example



The proposed heuristic is able to cover the square region using on average 17% less jammers than the uniform grid solution

Summary

Current accomplishments...

1. Developed several math. programming formulations
2. Formulations for deterministic and stochastic setup
3. Derived upper and lower bounds for uncertain case
4. Proof of convergence
5. Heuristic for uncertain case

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